

YEAR 12 Trial Exam Paper
2019
SPECIALIST MATHEMATICS

Written examination 1

Reading time: 15 minutes

Writing time: 1 hour

STUDENT NAME:

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring the following items into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 15 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions

- Write your **name** in the box provided above.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- You must answer the questions in English.

At the end of the examination

- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

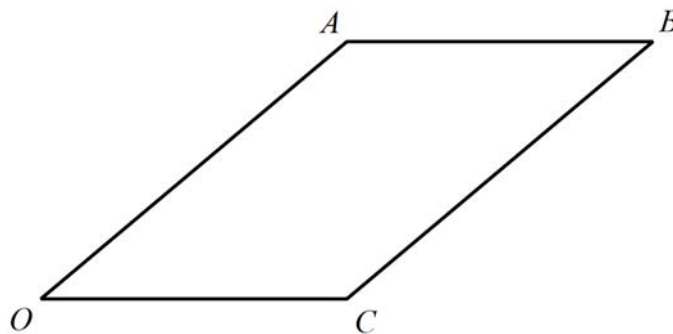
In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$.

Question 1 (4 marks)

Consider the parallelogram $OABC$ shown below where $\overrightarrow{OA} = m\mathbf{i} + \sqrt{39}\mathbf{k}$ and $\overrightarrow{OC} = -6\mathbf{i} + 8\mathbf{j}$.



- a. Find the possible values of m if $|\overrightarrow{OA}| = 2|\overrightarrow{OC}|$.

2 marks

- b.** Find the possible values of m if the angle between $\overrightarrow{OA}$ and $\overrightarrow{OC}$ is $\arccos\left(\frac{1}{5}\right)$. Give your answer in the form $\frac{\sqrt{a}}{b}$, where a and b are integers.

2 marks

Question 2 (3 marks)

Find the gradient of the curve with equation $2xy^2 + \arccos\left(\frac{x}{4}\right) - \frac{\pi xy}{3} = 1$ at the point $\left(2, \frac{1}{2}\right)$.

Give your answer in the form $\frac{\pi - a + \sqrt{a}}{b - c\pi}$, where a , b and c are integers.

Question 3 (4 marks)

Let $P(z) = z^3 - (a-1)z^2 + (b^2+4)z - 8$, $z \in \mathbb{C}$, where a and b are real constants.

Given that $P(1 - \sqrt{3}i) = 0$, determine all the solutions to $P(z) = 0$ and find the values of a and b .

Question 4 (4 marks)

Two types of chocolate blocks are selected and their masses are measured. Block X's masses are normally distributed with a mean of 175 grams and standard deviation of 3 grams. Block Y's masses are normally distributed with a mean of 165 grams and standard deviation of 4 grams. The distributions are independent of each other.

- a.** Find the approximate probability that the mass of a randomly selected block X chocolate is greater than the mass of a randomly selected block Y chocolate.

2 marks

- b.** A quality control officer suspects that the mean difference in mass of the two chocolate blocks is not 10 grams. The officer collects 100 samples of block X chocolate and 100 samples of block Y chocolate.

Calculate a 95% confidence interval for the mean difference in the mass of block X chocolate and block Y chocolate. Use an integer multiple of the standard deviation in your calculations.

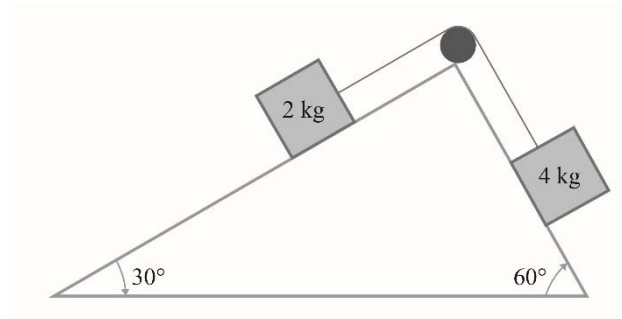
2 marks

Question 5 (3 marks)

Find $\tan(t)$ given that $t = \arcsin\left(\frac{4}{5}\right) - \arccos\left(\frac{5}{13}\right)$.

Question 6 (3 marks)

Two objects of masses 2 kg and 4 kg are attached by a light inextensible string that passes over a smooth pulley. The 2 kg mass is on a smooth plane inclined at 30° to the horizontal and the 4 kg mass is on a smooth plane inclined at 60° to the horizontal, as shown in the diagram below. Initially the pulley is locked, preventing the objects from moving. The pulley is unlocked and the 4 kg block accelerates down the plane.



- a.** Find the magnitude of the acceleration, $a \text{ ms}^{-2}$, of the 4 kg block.

2 marks

- b.** Determine the tension in the string connecting the two objects in newtons.

1 mark

Question 7 (4 marks)

A fish tank initially holds 200 L of water in which 10 kg of salt has been dissolved. Pure water then flows into the tank at a rate of 5 L per minute. The mixture is stirred continuously and flows out at a rate of 10 L per minute.

- a.** Show that the differential equation for x , the number of kilograms of salt in the tank after t minutes, is given by

$$\frac{dx}{dt} = \frac{-2x}{40-t}$$

1 mark

- b.** Solve the differential equation given in **part a.** to find x as function of t .

3 marks

Question 8 (4 marks)

The acceleration, $a \text{ ms}^{-2}$, of a motorcycle moving in a straight line relative to the origin with a velocity of $v \text{ ms}^{-1}$ is given by $a = v^3 + 16v$.

- a.** Given that $v = 0$ when $x = \frac{\pi}{16}$, where x is the displacement in metres, find an equation for the velocity of the motorcycle in terms of x .

3 marks

- b.** Hence, calculate the velocity, in ms^{-1} , of the motorcycle when $x = \frac{\pi}{8}$.

1 mark

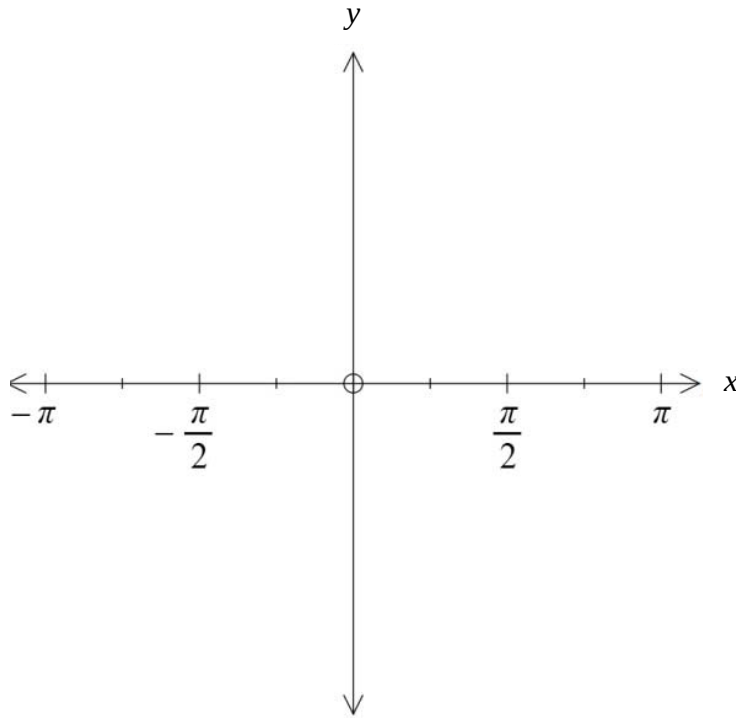
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Question 9 (6 marks)

Consider the function $f(x) = \arctan(2x - \pi)$.

- a. Sketch the graph of the function on the axes provided below, labelling any asymptotes with their equations and any intercepts with their coordinates.

2 marks



- b.** The function $f(x)$ is translated $\frac{\pi}{2}$ units to the left to form the function $y = g(x)$. The function $y = g(x)$ is then rotated about the y -axis.

Determine the volume of the solid generated when the region bounded by the graph of $y = g(x)$, the y -axis and the line $y = \frac{\pi}{3}$ is rotated about the y -axis. Give your answer in

the form $\frac{\pi}{a}(b\sqrt{c} - \pi)$, where a , b and c are integers.

4 marks

Question 10 (5 marks)

The position vector of a particle moving along a curve at time t seconds is given by

$$\mathbf{r}(t) = (2t^6 + 3)\mathbf{i} + \left(\frac{6}{7}t^7 + 1\right)\mathbf{j}, \quad t \geq 0, \text{ where distances are measured in metres.}$$

- a.** Show that the distance d that the particle travels along the curve in the first 2 seconds is given by

$$d = \int_0^2 6t^5 \sqrt{t^2 + 4} \, dt$$

1 mark

- $$d=3\left[au^{\frac{7}{2}}-bu^{\frac{5}{2}}+cu^{\frac{3}{2}}\right]_4^8$$

4 marks

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

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